

Physics 4B

Chapter 30: Induction and Inductance

"For every minute you are angry, you lose 60 seconds of happiness." – Ralph Waldo Emerson

"Consider how much more often you suffer from your anger and grief than from those very things for which you are angry and grieved." – Marcus Aurelius

"Holding on to anger is like grasping a hot coal with the intent of throwing it at someone else; you are the one who gets burned." – Buddha

Reading: pages 791 – 813

Outline:

- ⇒ introduction
 - Faraday's Law of induction
 - Lenz's Law
- ⇒ emf induced in a moving conductor
- ⇒ induced electric fields
 - Faraday's Law
- ⇒ inductors and inductance
 - inductance of a solenoid
 - self-induction
- ⇒ RL circuits
- ⇒ energy stored in a magnetic field
 - energy density of a magnetic field
- ⇒ mutual induction (read on your own)

Problem Solving Techniques

You should know how to compute the flux through a given area. In some cases, the magnetic field is uniform and you simply multiply the perpendicular component by the area. In other cases, the field is not uniform and you must carry out an integration of the perpendicular component over the area. In some problems, the magnetic field is given while in others the current that produces the field is given and you must use what you learned in the last chapter to write an expression for it as a function of position. For example, the field may be produced by current in a long straight wire.

Once an expression for the magnetic flux is found, you differentiate it with respect to time to find the emf around the boundary of the region. Look carefully at the expression for the flux and decide what is changing with time. It might be the magnitude of the magnetic field, the magnitude of the area of the region, or the orientation of the area with respect to the field.

In some problems, the boundary of the region is a conductor and you are asked to find the current in it. First, note all the emfs around the boundary. One of them is the emf associated with the changing

magnetic flux but there may be others, produced by batteries or generators, for example. Add them with their correct signs and divide the total by the resistance in the loop. Obviously, you must be very careful about the signs of the various emfs here.

Some problems ask for the electric field associated with a changing magnetic field. Use Faraday's

law in the form $\oint \vec{E} \cdot d\vec{s} = -d\Phi/dt$. All of the problems have cylindrical symmetry, with the electric field lines forming circles around the cylinder axis. For them, you integrate the tangential component of the electric field around one of the field lines, with result $\oint \vec{E} \cdot d\vec{s} = 2\pi rE$, where r is the radius of the circle. Find the rate of change of the magnetic flux through the circle and solve the Faraday's law equation for E .

Some problems ask you to use the definition of self-inductance ($L = N\Phi/i$) to compute one of the quantities that appear in it. To carry out this task, you may need to compute the magnetic flux Φ of the magnetic field, perhaps by carrying out an integration. Other problems deal with the emf generated by an inductor when the current changes. Use $\mathcal{E} = -L di/dt$.

Some problems deal with RL series circuits. If a source of emf, such as a battery, is in the circuit and the current is 0 at time $t = 0$ (because, for example, a switch is closed then), the current is given by

$i(t) = (\mathcal{E}/R)(1 - e^{-t/\tau_L})$, where the inductive time constant is $\tau_L = L/R$. If the current is i_0 at $t = 0$ and no source of emf is in the circuit, calculate $i_0 e^{-t/\tau_L}$.

You may be asked to compute the potential difference across the resistor (iR) or across the inductor ($L di/dt$). Be sure you can tell which end is at the higher potential. You may also be asked for the rate with which the source of emf is supplying energy ($i\mathcal{E}$), the rate at which the resistor is dissipating energy (i^2R), or the rate at which the inductor is storing energy ($Li di/dt$). If you are asked for the

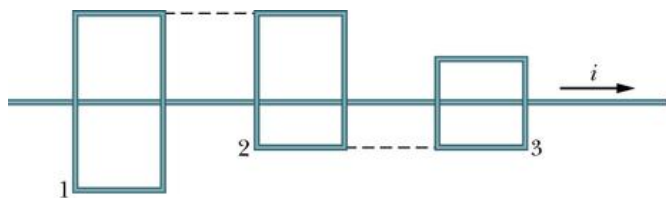
energy densities use $u_E = \frac{1}{2} \epsilon_0 E^2$ and $u_B = B^2/2\mu_0$.

Some problems deal with the calculation of mutual inductance. Here you assume a current in one circuit and compute the magnetic flux it produces in another circuit.

Questions and Example Problems from Chapter 30

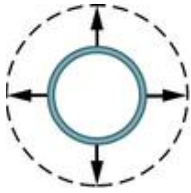
Question 1

In the figure below, a long straight wire with current i passes (without touching) three rectangular wire loops with edge lengths L , $1.5L$, and $2L$. The loops are widely spaced (so as to not affect one another). Loops 1 and 3 are symmetric about the long wire. Rank the loops according to the size of the current induced in them if current i is (a) constant and (b) increasing, greatest first.



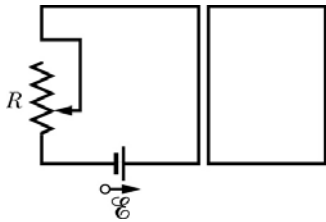
Question 2

If the circular conductor in the figure below undergoes thermal expansion while it is in a uniform magnetic field, a current will be induced clockwise around it. Is the magnetic field directed into the page or out of it?



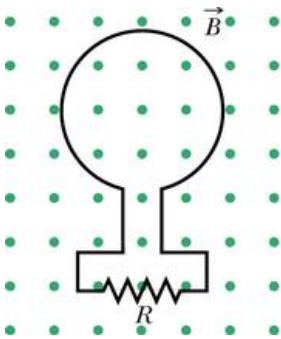
Question 3

If the variable resistance R in the left-hand circuit of the figure below is increased at a steady rate, is the current induced in the right-hand loop clockwise or counterclockwise?



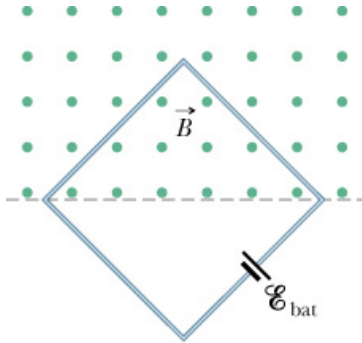
Problem 1

In the figure below, the magnetic flux through the loop increases according to the relation $\Phi_B = 6.0t^2 + 7.0t$, where Φ_B is in milliwebers and t is in seconds. (a) What is the magnitude of the emf induced in the loop when $t = 2.0$ s? (b) Is the direction of the current through R to the right or left?



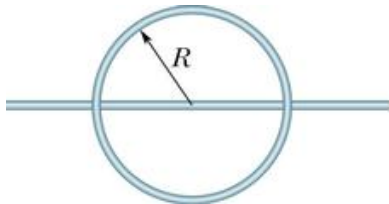
Problem 2

A square wire loop with 2.00 m sides is perpendicular to a uniform magnetic field, with half the area of the loop in the field as shown in the figure below. The loop contains an ideal battery with emf $\mathcal{E} = 20.0 \text{ V}$. If the magnitude of the field varies with time according to $B = 0.0420 - 0.870t$, with B in teslas and t in seconds, what are (a) the net emf in the circuit and (b) the direction of the current through the battery?



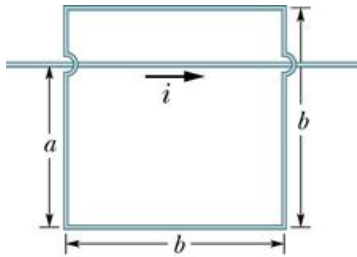
Problem 3

In the figure below, a wire forms a closed circular loop, with radius $R = 2.0 \text{ m}$ and resistance 4.0Ω . The circle is centered on a long straight wire; at time $t = 0$, the current in the long straight wire is 5.0 A rightward. Thereafter, the current changes according to $i = 5.0 \text{ A} - (2.0 \text{ A/s}^2)t^2$. (The straight wire is insulated, so there is no electrical contact between it and the wire of the loop.) What are the magnitude and direction of the current induced in the loop at times $t > 0$?

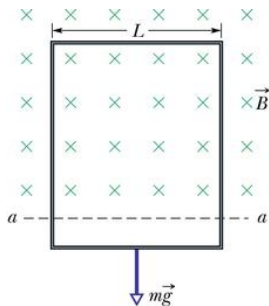


Problem 4

For the situation shown in the figure below, $a = 12.0$ cm and $b = 16.0$ cm. The current in the long straight wire is given by $i = 4.50t^2 - 10.0t$, where i is in amperes and t is in seconds. (a) Find the emf in the square loop at $t = 3.00$ s. (b) What is the direction of the induced current in the loop?

**Problem 5**

In the figure below, a long rectangular conducting loop, of width L , resistance R , and mass m , is hung in a horizontal, uniform magnetic field that is directed into the page and that exists only above line aa . The loop is then dropped; during its fall, it accelerates until it reaches a certain terminal speed v_t . Ignoring air drag, find that terminal speed.



Problem 6

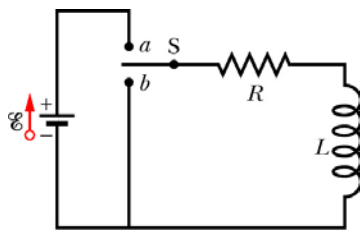
The inductance of a close-packed coil of 400 turns is 8.0 mH. Calculate the magnetic flux through the coil when the current is 5.0 mA.

Problem 7

A battery is connected to a series RL circuit at time $t = 0$. At what multiple of τ_L will the current be 0.100% less than its equilibrium value?

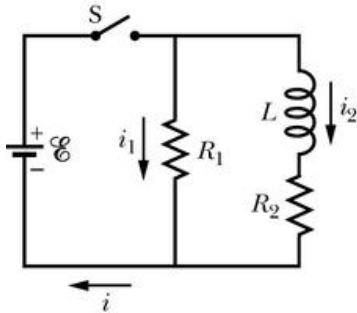
Problem 8

Consider the RL circuit of the figure below. In terms of the battery emf \mathcal{E} , (a) what is the self-induced emf \mathcal{E}_L when the switch has just been closed on a, and (b) what is \mathcal{E}_L when $t = 2.0\tau_L$? (c) In terms of \mathcal{E}_L , when will \mathcal{E}_L be just one-half the battery emf \mathcal{E} ?



Problem 9

In the figure below, the battery is ideal, $\mathcal{E} = 10 \text{ V}$, $R_1 = 5.0 \ \Omega$, $R_2 = 10 \ \Omega$, and $L = 5.0 \text{ H}$. Switch S is closed at time $t = 0$. Just afterwards, what are (a) i_1 , (b) i_2 , (c) the current i_s through the switch, (d) the potential difference V_2 across resistor R_2 , (e) the potential difference V_L across the inductor L , and (f) the rate of change di_2/dt ? A long time later, what are (g) i_1 , (h) i_2 , (i) i_s , (j) V_2 , (k) V_L , and (l) di_2/dt ?



Problem 10

A coil with an inductance of 2.0 H and a resistance of 10Ω is suddenly connected to an ideal battery with $\mathcal{E} = 100 \text{ V}$. At $t = 0.10 \text{ s}$ after the connection is made, what is the rate at which (a) energy is being stored in the magnetic field, (b) thermal energy is appearing in the resistance, and (c) energy is being delivered by the battery?

Problem 11

Prove that, after switch S in the figure to the right has been thrown from a to b , all the energy stored in the inductor will ultimately appear as thermal energy in the resistor.

